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Mathematical Modeling of a Ship Motion in Waves under Coupled Motions

Aung Myat Thu, Ei Ei Htwe, Htay Htay Win

Abstract— **In this paper, research area is emphasized on motion of a ship due to anti- symmetric coupled motion of roll-yaw and sway-roll-yaw. This paper also expressed wave forces, hydrodynamic coefficients and 2-DOF and 3-DOF model of a surface vessel and its motions. The required hydrodynamic coefficients and wave forces are obtained from integration of sectional added-mass, damping and restoring coefficients, derived from Frank's close-fit conformal mapping method. The governing equations for motion equations are solved numerically by using Runge-Kutta method. The vessel used in this analysis is monohull vessel type which is 28 m long. In the calculation of floating body motions, two-dimensional added mass, damping, and excitation for each ship sections are solved out. The governing equations comprised of second order ordinary differential equation that come from equation of hydrodynamic forces and external exciting forces. This paper describes the complete model of Tug and its numerical calculation solves with Matlab.**

Keywords **—Hydrodynamic Coefficients, Added-mass, Damping, Coupled Motion, Strip Theory**

I. Introduction

Coupled motions are the most studied among the six degrees of freedom of a ship in a seaway. The predictions enjoy a high level of accuracy that is not equaled by predictions for the other degrees of freedom. The most popular method for the analytical predictions of the motions of ship is the strip theory. The study of wave-induced loads and motions of ships is important both in ship design and operational studies. Early research in ship hydrodynamics is developed primarily in calm water conditions. To predict the hydrodynamic loads and motions of the ship, model test, full-scale trials, or numerical calculations may be used. Based on different approaches to solutions, the numerical technique in ship motion analysis can be categorized as frequency domain and time domain methods. When solving the equations of motion, one may use either the frequency domain method or the time domain method. [4]

For a floating body with lateral symmetry in shape and weight distribution, the six coupled equations of motion can be reduced to two sets of equations, where the first set consisting of surge, heave, and pitch can be decoupled from the second set consisting of sway, roll, and yaw. The investigation of the second set since the roll motion is important with respect to the stability of the floating body.

In the past, many researchers have analyzed the effects of nonlinear restoring moments of a rolling ship. Kristian Koskinen [2012] published the numerical simulation of ship motion due to waves and maneuvering. In the report shows the reasons for the development of a ship simulation model in five degree of freedom implemented in MATLAB script. The results were agreed with the linear method, three degree of freedom non-linear models as well as with the experimental results.

R. A. Ibrahim [2009] presented modeling of ship roll dynamics and its coupling with heave and pitch. The nonlinear dynamic modeling of ship motions in roll, pitch, and heave has been formulated based on physical ground. One can use the coupled nonlinear equations motion to examine only the ship motion in roll oscillations under regular and random sea waves.

Theoretical studies on low order potential damping models for surface vessels was carried out by Kari Unneland and Thor I. Fossen. In the research the modeling of the radiation forces in a 3-DOF model of a surface vessel for DP has been presented; special attention was given to the representation of potential damping forces in state-space form. A novel passivity preserving reduction technique is proposed and compared with existing techniques. This approach taken better care of the coupled dynamics and in addition the order of the model can be reduced significantly compared to the SISO-representation.

Tristan Perez and Thor I. Fossen (2006) presented a detailed simulation model of a naval coastal patrol vessel in a 4-DOF Simulink model of a coastal Patrol vessel for maneuvering in waves. This model has been implemented in Matlab-Simulink. The wave excitation forces are simulated as a multisine time series. This uses the force frequency response functions (FRF) of the vessel in combination with the wave spectrum. Three simulation models were carried out during two maneuvers. The first model shows the vessel performing a 20-20 zig-zag test in calm water. And the second was about the ship sailing northwards under the influence of waves, and the last one verified the wave excitation forces acting on the ship during this transit. A p-controller autopilot was applied to achieve this.

J.-S. Wu (1996) was carried out an exact solution for a simplified model of the heave and pitch motions of a ship hull due to a moving load and a comparison with some experimental results. The author had worked out to investigate the coupled heave and pitch motions of a simplified non-uniform ship hull floating on a still water surface and subjected to a moving load.

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II. The Vessel

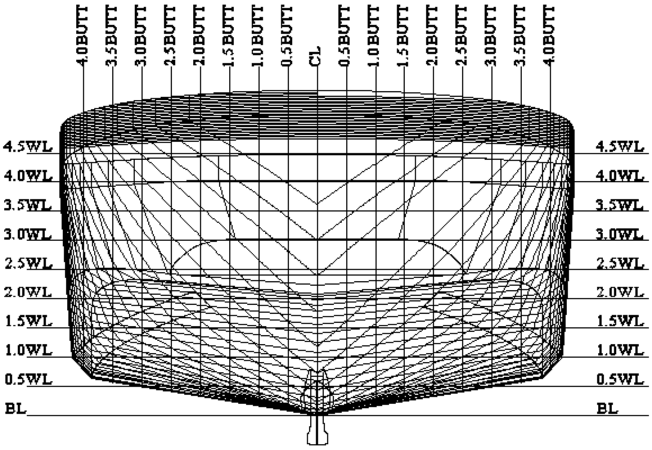

Figure (1) Body Plan of 1000 HP Twin Screw Tug

The vessel model presented in this paper corresponds to a small relatively fast ship. Figure (1) shows the vessel profile. This vessel is a 1000 HP screw Tug which is currently serving in Myanmar. The mathematical model presented in this paper is based on the drawing lines and load conditions. The vessel's main particulars and the loading condition are summarized in Table.1. The authors have modified the data of 1000 HP Tug limited so as to match the load condition, such that the seakeeping model can be combined with the maneuvering model to give a realistic simulation environment.

Table.1. Hydrodynamics Characteristics of Tug [9]

Parameter	Symbols	Value	Unit
Length Overall	L_{OA}	29	m
Breath	B	8	m
Draught	T	5.5	m
Draft	T	2.643	m
Volume	V	222.04	m^3
Displacement	Δ	227.10	tons
GM	GM	1.093	m
LCF	LCF	12.078	m
LCB	LCB	13.688	m

III. MATMATICAL FORMUALTION

A coupled roll-yaw mathematical model as followed based on a nonlinear strip theory is used to calculate yaw and roll motions in regular head seas with parametric rolling taken into account. In order to construct the governing equations of motion, the following assumptions are made.

(i) The floating body is slender and rigid with symmetric distribution of mass.
(ii) Motion amplitude is small so that equations can be linearized.
(iii) Except in roll motion, the effect of viscosity is neglected.
(iv) Incident waves are unidirectional and of single periodicity.
(v) Due to lateral symmetry, longitudinal and transverse motions are decoupled.

The six motions of ship and its center of gravity G has been defined in the Figure 2.

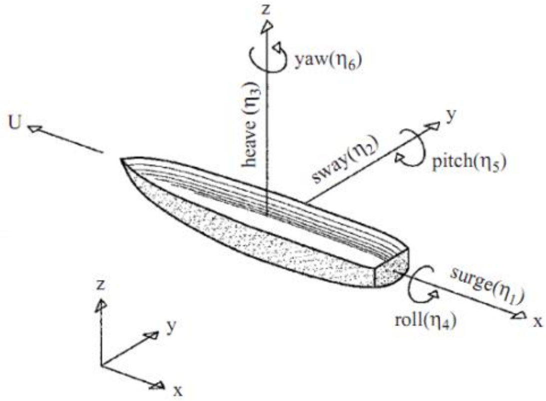

Figure (2) Six Degree of Freedom Motion of a Ship[18]

According to Newton's second law, the equations of motion for six degrees of freedom of an oscillating ship in waves in an earth-bound axes system have to be written as follows[11]:

$$\sum_{j=1}^{6}\{M_{ij}.\ddot{x}_i\} = F_i \quad \text{for i= 1,…6} \qquad (1)$$

Because a linear system has been considered in this condition, the forces and moments in the right-hand side of these equations consist of a superposition:

i. so-called hydromechanics forces and moments, caused by harmonic oscillations of the rigid body in the undisturbed surface of a fluid being previously at rest, and
ii. So-called exciting wave forces and moments on the restrained body, caused by the incoming harmonic waves.

With this, the system with six degrees of freedom moving ship in waves can be considered as a linear mass-damping-spring system with frequency dependent coefficients and linear exciting wave forces and moments.

$$\sum_{j=1}^{6}\{(M_{ij}+a_{ij}).\ddot{x}_i + b_{ij}.\dot{x}_i + c_{ij}.x_i\} = F_i \qquad (2)$$

for: i= 1,…6

In here, x_i with indices i=1,2,3 are the displacements of G (surge, sway and heave) and x_i with indices i=4,5,6 are the rotations about the axes through G (roll, pitch and yaw). The terms with indices ij present for motion i the coupling with motion j [11].

The masses in these equations of motion consist of solid masses or solid mass moments of inertia of the ship (M_{ij}) and "added" masses or "added" mass moments of inertia caused by the disturbed water, called hydrodynamic masses or mass moments of inertia (a_{ij}).

An oscillating ship generates waves itself too, since energy will be radiated from the ship. The hydrodynamic damping terms ($b_{ij}\ x_i$) account for this. For the heave, roll and pitch motions, hydrostatic spring terms ($c_{ij}\ x_i$) have to be added. The right hand sides of these equations of motion consist of the exciting wave forces and moments (F_i).

$$[M_{ij}] = \begin{bmatrix} M & 0 & 0 \\ 0 & I_4 & -I_{46} \\ 0 & -I_{46} & I_6 \end{bmatrix}$$

$$[a_{ij}] = \begin{bmatrix} A_{22} & A_{24} & A_{26} \\ A_{42} & A_{44} & A_{46} \\ A_{62} & A_{64} & A_{66} \end{bmatrix}$$

$$[b_{ij}] = \begin{bmatrix} B_{22} & B_{24} & B_{26} \\ B_{42} & B_{44} & B_{46} \\ B_{62} & B_{64} & B_{66} \end{bmatrix}, [C_{ij}] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & C_{44} & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\dot{x}_{ij} = \begin{bmatrix} \ddot{\eta}_2 \\ \ddot{\eta}_4 \\ \ddot{\eta}_6 \end{bmatrix}, \dot{x}_{ij} = \begin{bmatrix} \dot{\eta}_2 \\ \dot{\eta}_4 \\ \dot{\eta}_6 \end{bmatrix}, x_{ij} = \begin{bmatrix} \eta_2 \\ \eta_4 \\ \eta_6 \end{bmatrix}, a_{ij} = \begin{bmatrix} F_2 \\ F_4 \\ F_6 \end{bmatrix}$$

$$\bar{F}_2 = \alpha\rho\int(f_2 + h_2)dx, \bar{F}_4 = \alpha\rho\int(f_4 + h_4)dx$$

$$\bar{F}_6 = \alpha\rho\int x(f_2 + h_2)dx, F_i = \bar{F}_i \sin(\omega t), i = 2,4,6$$

Using Equations (1) in (2), the governing equations for roll-yaw motions can be written as

$$(A_{44} + I_4)\ddot{\eta}_4 + B_{44}\dot{\eta}_4 + C_{44}\eta_4 + (A_{46} - I_6)\ddot{\eta}_6 + B_{46}\dot{\eta}_6 = F_4 \quad (3)$$

$$(A_{64} + I_{46})\ddot{\eta}_4 + B_{64}\dot{\eta}_4 + (A_{66} - I_6)\ddot{\eta}_6 + B_{66}\dot{\eta}_6 = F_6 \quad (4)$$

Using Equations (1) in (2), the governing equations for sway-roll-yaw motions can be written as

$$(A_{22} + M)\ddot{\eta}_2 + B_{22}\dot{\eta}_2 + (A_{24} - M)\ddot{\eta}_4 + B_{24}\dot{\eta}_4 + B_{26}\ddot{\eta}_6 + B_{26}\dot{\eta}_6 = F_2 \quad (5)$$

$$(A_{42} + M)\ddot{\eta}_2 + B_{42}\dot{\eta}_2 + (A_{44} - I_4)\ddot{\eta}_4 + B_{44}\dot{\eta}_4 + C_{44}\eta_4 + (A_{46} - I_{46})\ddot{\eta}_6 + B_{46}\dot{\eta}_6 = F_4 \quad (6)$$

$$\ddot{\eta}_2 + B_{62}\dot{\eta}_2 + (A_{64} - I_{46})\ddot{\eta}_4 + B_{64}\dot{\eta}_4 + (A_{66} + I_6)\ddot{\eta}_6 + B_{66}\dot{\eta}_6 = F_6 \quad (7)$$

The equations of motion in 2-DOF and 3-DOF provided in equation (3) through (7) can now be solved numerically [11]. To solve the second order differential motion equations, the equations can be reduced to the first order equation. These equations may be included in a function and called from an ODE solver.

3.1. Calculation of Wave Spectra

Irregular ocean waves are often characterized by a "wave spectrum", this describes the distribution of wave energy (height) with frequency. The continuous frequency domain representation shows the power density variation of the waves with frequency and is known as the wave amplitude energy density spectrum.

$$S(\omega) = \frac{A}{\omega^5} e^{-B/\omega^4} \quad (8)$$

$$B = 1.25\,\omega_m^4, A = 4BE_s, E_s = {H_{1/3}^2}/{16}$$

The transfer function or response function for roll motion in frequency domain can be written as follow [13]:

$$J\theta'' + B_{44}r^2\theta' + B_{44}r^2\theta = (F_{4I} + F_{4D})r\,\eta_4 + F_{4R}.r^2\theta$$

$$\frac{\theta(\omega)}{\eta_4} = \frac{(F_{4I} + F_{4D})r}{-(F_{4R}.r^2) - \omega^2 J + j\omega B_{44}r^2 + C_{44}r^2}$$

Table.2 Sectional Coefficients of the Floating Body

Wave	Two Dimensional Coefficients							
ω	a_{22}	a_{44}	a_{24}	b_{22}	b_{44}	b_{24}	fh2	fh4
0.56	1.6	0.07	-0.25	0.6	0.01	-0.07	2.25	1.9
0.74	0.65	0.055	-0.13	1.0	0.02	-0.16	1.5	1.2
1.24	0.05	0.035	-0.02	0.7	0.012	-0.1	0.34	0.28

IV. Results And Discussion

In order to illustrate coupling motions of rolling-yaw and swaying-rolling-yawing, Equations (3), (4) and (5), (6), (7) are evaluated numerically. To simulate the motion of ship required frequency dependent hydrodynamic coefficients, related to sectional added mass, damping and wave exciting force are adopted from the experimental results conducted by Vugts [19] and close-fit curve given by Frank and Salvesan [18] shown in table (2). Figure 3(1-a) to 3(3-b) illustrate the ship floating in wave in couple motion Roll-Yaw under three encounter wave frequencies. The inspection of numerical solution shows that the nature of ship motions is similar but different in magnitude.

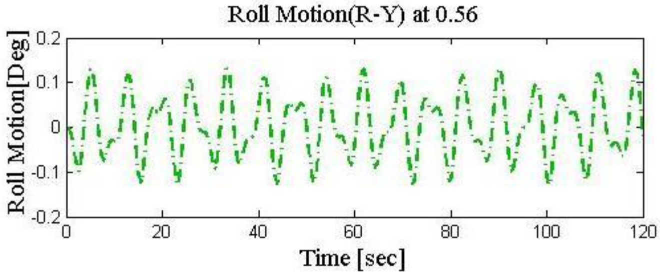

Figure 3(1-a)

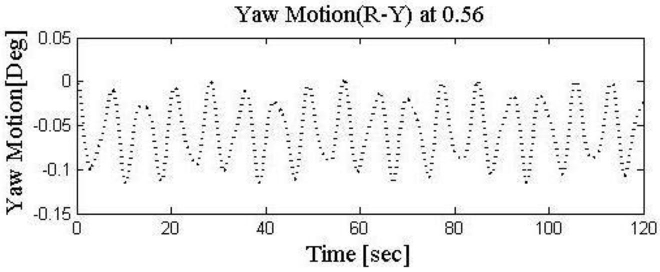

Figure 3(1-b)

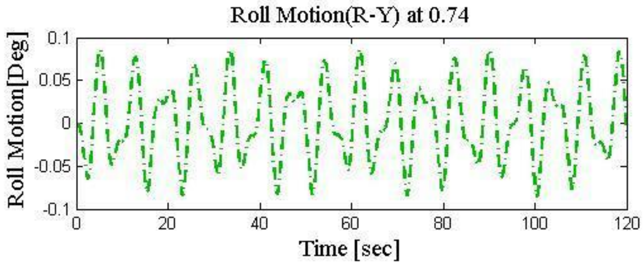

Figure 3(2-a)

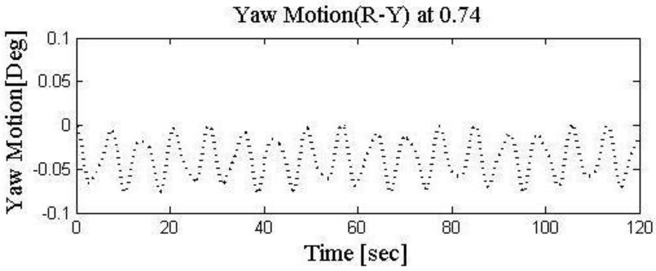

Figure 3(2-b)

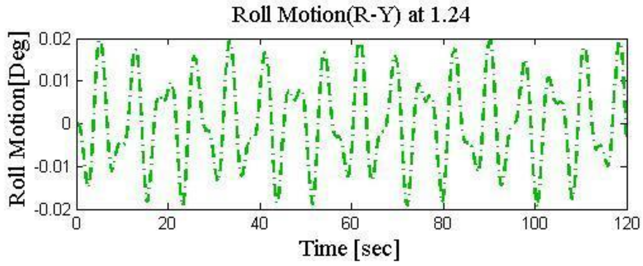

Figure 3(3-a)

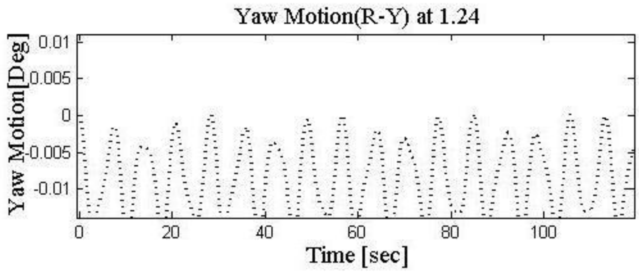

Figure 3(3-b)

The time history of motion in coupled sway-roll-yaw motion are also analyze in the same way. The time history of motion for frequencies 0.56 rad/s, 0.74 rad/s, and 1.24 rad/s are shown in Figure 4(1-a) through 4(3-c). It can be observed that the sway displacement is largest at wave frequency 1.24 rad/s. The maximum sway displacement corresponding to these frequencies are found to be 10 m, 5 m, and 2.8 m, respectively. The same phenomena occur for roll motion with highest motion at frequency 1.24 with 30 degree and lower in other frequencies. But these roll displacements (in degree) are apparently larger in every case for coupled roll-yaw motions. But in coupled roll-yaw motion the lowest frequency 0.56 have larger frequency with 0.14 degree. It also can be observed that the roll displacement is maximum for wave frequency 0.56 rad/s and it decreases as wave frequency increases. The characteristics of yaw motion are shown in Figure 3(1-c), 3(2-c) and 3(3-c) for sway-roll-yaw coupled condition. Figure 2(1-b), 2(2-b) and 2(3-b) exhibit the characteristics of yaw motion in roll-yaw condition. The maximum yaw displacement corresponding to three frequencies are found to be 10m, 20, and 23 (in degree). These motion histories also remain higher than in Roll-Yaw condition.

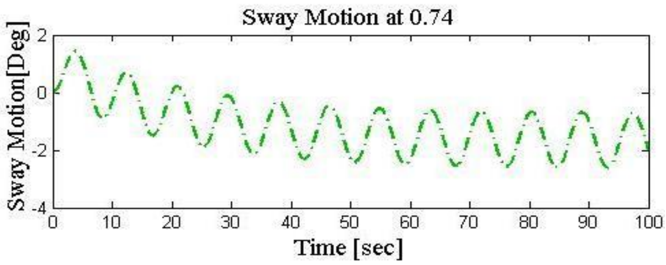

Figure 4(1-a)

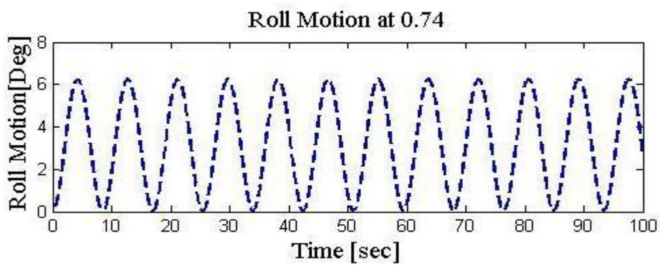

Figure 4(1-b)

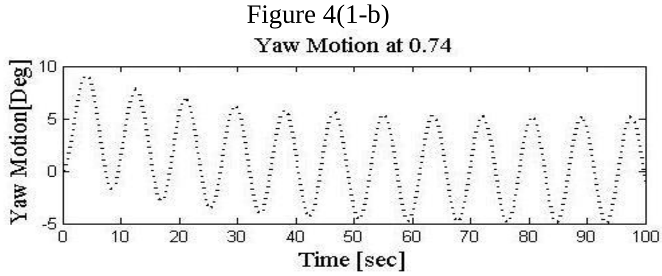

Figure 4(1-c)

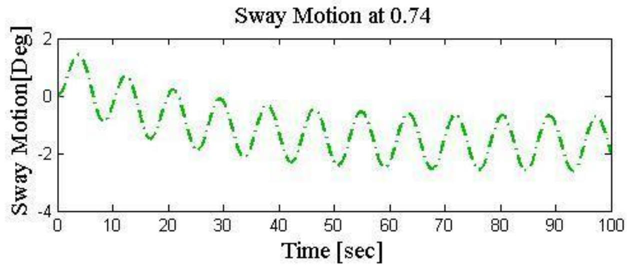

Figure 4(2-a)

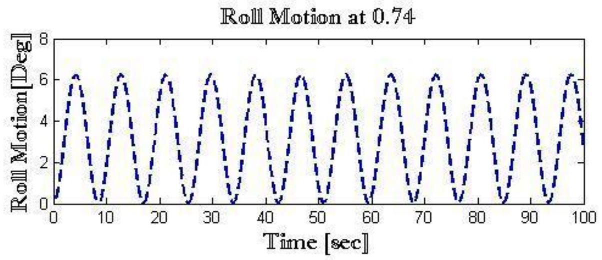

Figure 4(2-b)

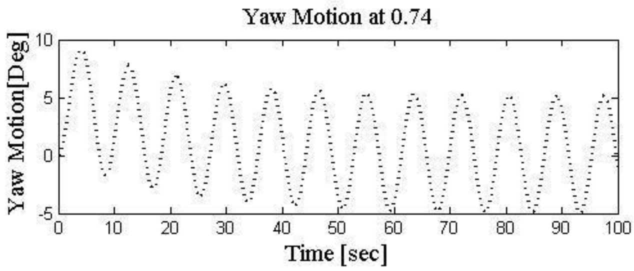

Figure 4(2-c)

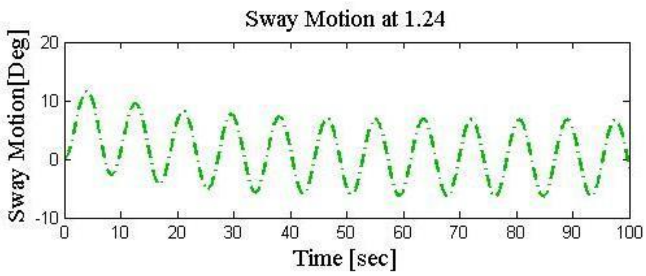

Figure 4(3-a)

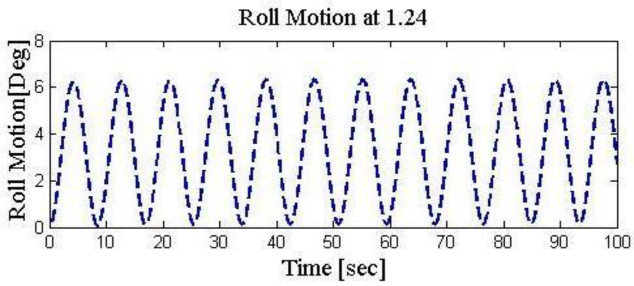

Figure 4(3-b)

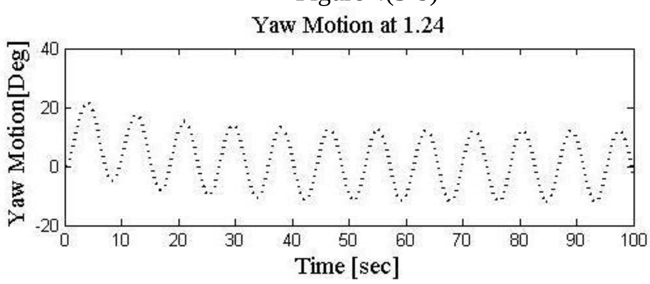

Figure 4(3-c)

To investigate the wave encounter spectrum and transfer function for roll motion, the analysis is carries out with various wave heights at sea state 2 to 6. There also can be seen that encounter wave frequency is between in the range 0~2 rad/s. The spectrums with different sea state are shown in the Fig.4.

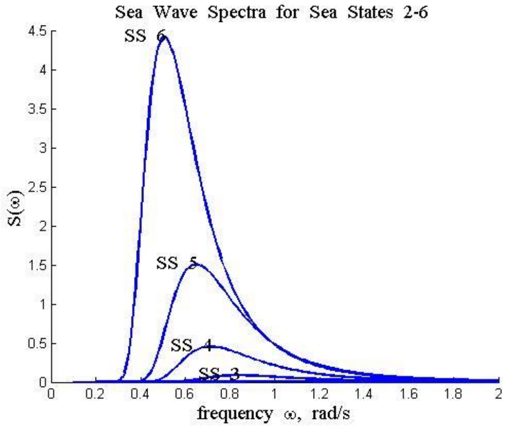

Fig.5 Wave Spectrum

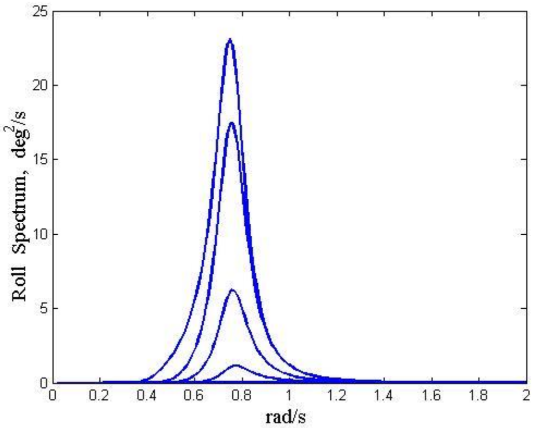

Fig.6. Roll Transfer Function

The transfer function for roll motion is shown in Fig.5. There also can be seen that the value of the transfer function goes towards peak from 0.4 to 1 wave frequency. This indicates that the calculations are realistic, or at least physical. For the lower sea state wave lengths high waves do not exist, so it makes sense that the excitation force approaches zeros as the frequency increases. For higher sea state, the ship will follow the wave motion, so the forces are hydrostatic or quasi-steady. This means that the wave excitation force should balance the restoring terms in the roll equation of motion.

V. Conclusion

This research area mainly focus on the analysis for coupled roll-yaw and sway-roll-yaw motion when encounter different wave frequency to a ship. For roll motion, motion of coupled sway-roll-yaw is significantly higher than coupled roll-yaw motion with the variation of time series. For the yaw motion, the motion variation in coupled sway-roll-yaw is obviously higher than the motion in roll-yaw. Although the yaw motion has occurred in homogenous form, the roll motion can be seen as non homogenous form. One of the main obvious thing from the results is that condition of ship in every coupled motions are leading resonance condition that mainly caused by wave encounter frequency. It can conclude that condition of unstable condition in motion will happen when sway motion are considered in coupled roll-yaw motion.

Appendix

The roll restoring coefficient C_{44} arising in Eq.3 is given by

$C_{44} = \rho g \nabla \overline{GM}$,
$I_{44} = k_{xx}^2 . \rho . \nabla$,
$I_{66} = k_{zz}^2 . \rho . \nabla$
$k_{xx} = 0.30. B \ to \ 0.4. B$,
$k_{zz} = 0.22. L \ to \ 0.28. L$

$$A_{22} = \int_0^L a_{22}. dx - \frac{U}{\omega_e^2} b_{22}^A,$$

$$A_{24} = \int_0^L a_{24}. dx - \frac{U}{\omega_e^2} b_{24}^A$$

$$A_{26} = \int_0^L x. a_{22}. dx + \frac{U}{\omega_e^2} \int_0^L b_{22}. dx - \frac{U}{\omega_e^2} x_a b_{22}^A + \frac{U}{\omega_e^2} a_{22}^A = A_{62}$$

$$A_{46} = \int_0^L x. a_{24}. dx + \frac{U}{\omega_e^2} \int_0^L b_{24}. dx - \frac{U}{\omega_e^2} x_a b_{24}^A + \frac{U}{\omega_e^2} a_{24}^A$$

$B_{22} = \int_0^L b_{22}. dx - U \frac{U}{\omega_e^2} . a_{22}^A$, $B_{44} = \int_0^L b_{44}. dx - U. a_{44}^A$

$$A_{64} = \int_0^L x. a_{24}. dx + \frac{U}{\omega_e^2} \int_0^L b_{24}. dx - \frac{U}{\omega_e^2} x_a b_{24}^A$$

$$A_{66} = \int_0^L x^2. a_{22}. dx + \frac{U}{\omega_e^2} \int_0^L b_{22}. dx - \frac{U}{\omega_e^2} x_a b_{22}^A + \frac{U^2}{\omega_e^2} x_a a_{24}^A$$

$$B_{46} = \int_0^L x. b_{24}. dx - U \int_0^L a_{24}. dx - U. x_a. b_{24}^A + \frac{U}{\omega_e^2} . b_{24}^A$$

$$B_{64} = \int_0^L x. b_{24}. dx + U \int_0^L a_{24}. dx + U. x_a. a_{24}^A$$

$$B_{66} = \int_0^L x^2. b_{22}. dx + \frac{U^2}{\omega_e^2} \int_0^L b_{22}. dx - U. x_a. a_{22}^A + \frac{U^2}{\omega_e^2} x_a b_{24}^A$$

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